SIMULATION OF TURBULENT CHANNEL FLOW USING A VISCOELASTIC FLUID MODEL BASED ON THE SQUARE-ROOT TRANSFORMATION FOR THE CONFORMATION TENSOR

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INTRODUCTION

The dilution of an infinitesimal amount of flexible polymers in certain organic solvents leads to a significant turbulent drag reduction. Direct numerical simulations (DNS) appear as a powerful stencil for the analysis of the interactions between turbulence and polymer chains. Aiming to take into account the polymers diluted in a Newtonian solvent the Finite Extensible Nonlinear Elastic model in the Peterlin approximation (FENE-P) is largely used. This model is based on the evolution equations of the symmetric positive definite (SPD) conformation tensor, which represents the spatial configuration of the polymeric chains.
However, in wall-bounded turbulent flows, the conformation tensor may lose its SPD property, leading to numerical instabilities, particularly for computational codes using high-order spectral methods. In order to overcome this limitation, Sureshkumar *et al.* [1] proposed the addition of an artificial diffusion term in the conformation tensor equation. Since this term is note relevant at the simulated scales, it should be as small as possible.
During the last decade, some alternatives without the need of an artificial diffusion term have been proposed. Fattal and Kupferman [2] proposed an evolution equation based on a logarithm transformation for the conformation tensor, which guarantees the SPD property of the conformation tensor. Vaithianathan *et al.* [3] suggested that the numerical instabilities preventing time marching for the conformation tensor come from the convection terms in its evolution equation. Hence, the authors introduced a numerical scheme limiting the convective fluxes, and respecting the SPD property of the conformation tensor. More recently, Balci *et al.* [4] presented a method based on an evolution equation for the square-root of the conformation tensor. Such methodology also ensures mathematically the SPD property of the conformation tensor – although it seems to be simpler and computationally cheaper compared to the previous methods. Nevertheless, to the best of our knowledge, this last method has only been tested in academic 2D and non-shear flows, with negligible advection.
In the present work, we propose the implementation of the square-root transformation in a 3D computational code [5,6] with quasi-spectral precision considering turbulent drag-reducing channel flows. The efficiency of the new code will be compared to the standard conformation tensor-based code. Currently, the latter makes use of an artificial diffusion term which is adjusted to maintain the SPD property of the conformation tensor in at least 99% of the domain. In particular, we are evaluating the need (or not) of maintaining the artificial diffusion term in the constitutive equation of the square-root conformation tensor in channel flows. We also intend to enrich our DNS channel flow database [7].

METHODOLOGY

The square-root transformation proposed by Balci *et al.* [2] applies for some viscoelastic (conformation tensor-based) models such as Oldroyd-B and FENE-P. Their proposal is to reformulate such models in terms of the square-root tensor of the conformation tensor, $\boldsymbol{c}$, which verifies the relation $\boldsymbol{c} = \boldsymbol{bb}$, where $\boldsymbol{b}$ is the (*a priori* SPD) square-root tensor of **c**. For the FENE-P model, its evolution equation has the form

$$\frac{\partial \boldsymbol{b}}{\partial t} + \boldsymbol{u}\nabla\boldsymbol{b} - \boldsymbol{ab} - \boldsymbol{b}\nabla\boldsymbol{u} + \frac{1}{2We_h}\left[\left(\frac{L^2-3}{L^2-\{\boldsymbol{c}\}}\right)\boldsymbol{b} - \boldsymbol{b}^{-1}\right] = 0 \quad , \tag{1}$$

where $\boldsymbol{u}$ is the velocity vector, L is the maximum extensibility of the polymer chain, $\{\boldsymbol{c}\}$ is the trace of the conformation tensor, $\boldsymbol{b}^{-1}$ is the inverse of the square-root tensor. The friction Weissenberg number is $We_h = \lambda u_\tau / h$, where λ is the relaxation time of the polymer, u_τ is the wall friction velocity, and h denotes the channel half gap. The tensor $\boldsymbol{a}$ is a skew-symmetric tensor obtained by the relation $\boldsymbol{r} = \boldsymbol{ab} + \boldsymbol{b}\nabla\boldsymbol{u}$, so that tensor $\boldsymbol{r}$, and thus the evolution equation (1), be symmetric. Hence, given a symmetric initial field of $\boldsymbol{b}$, such choice enables preserving its symmetry.

RESULTS AND DISCUSSION

The first attempts to run turbulent channel simulations using Eq. 1, i.e. the square-root transformation without any artificial diffusion, have not converged. We noticed however that the method is stable for steady laminar flows. We believe that this is due to large scale instabilities associated to wall-bounded turbulent shear flows. Therefore, analogously

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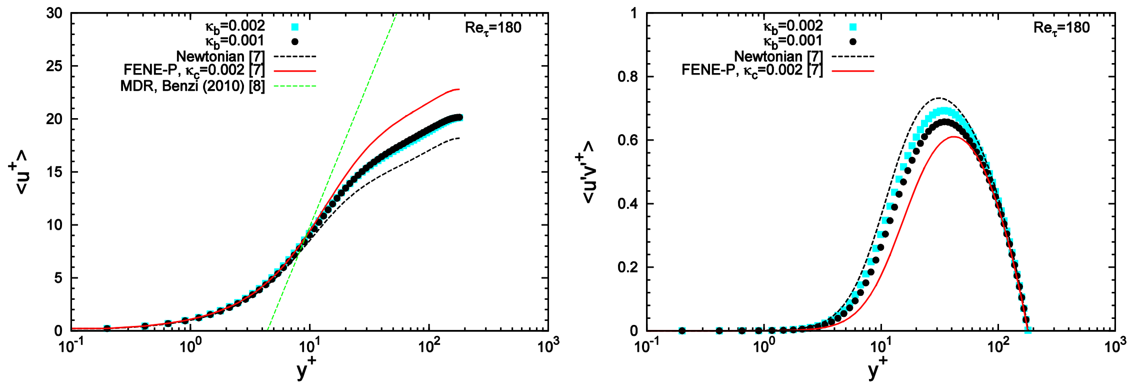
to Sureshkumar and co-authors' [1] proposition, we introduced an artificial diffusion term which is proportional to the Laplacian of the tensor $\boldsymbol{b}$. The proportionality constant is the artificial diffusivity κ_b. Theoretically, the artificial diffusivity of the conformation tensor should be naturally divided by two when the square-root transformation is applied. The goal here is to make the artificial diffusivity as small as possible.

Results

We present here results for simulations at friction Reynolds number $Re_\tau = u_\tau h/\nu_0$ equal to 180, where ν_0 applies for the zero shear rate total kinematic viscosity, friction Weissenberg number $We_\tau = \lambda u_\tau{}^2/\nu_0$ equal to 50 and L=30. Two values of artificial diffusivity coefficient, κ_b=2x10^{-3} and 1x10^{-3}, are being considered. In both cases, the flow is still about to achieve a fully statistical steady state (steady mean flow) and we emphasize that, for now, we only analyse tendencies. Even so, we can note already some coherent trends when comparing the present preliminary results to Thais and co-authors' [5-6] FENE-P data [7] available for a similar case considering the standard conformation tensor-based approach. The mean velocity profiles and the main shear Reynolds stress ($\overline{u'v'}$) are shown in Figs. 1a and 1b, respectively.



(a) (b)

Figure 1. Results for mean velocity profile (a) and main shear Reynolds stress (b) at Re_τ=180. All viscoelastic simulations were performed with We_τ=50 and L=30. Newtonian and FENE-P (κ_c=2x10^{-3}) data are from [7]. Maximum drag reduction (MDR) ($u^+ = 12 \ln y^+ - 17.8$) in (a) is from the theory proposed by Benzi [8].

Both Newtonian and FENE-P results in Figs. 1a and 1b are available in the DNS database [7]. The FENE-P simulations were also performed with an artificial diffusion term in the conformation tensor equation and the artificial diffusivity used was κ_c=2x10^{-3}.

FINAL REMARKS AND FURTHER WORK

We implemented a method [2] based on a square-root transformation to the conformation tensor using the FENE-P model to be applied to turbulent drag-reducing channel flows. Such method guarantees the symmetric positive definiteness of the conformation tensor.

The results suggest the need, for this kind of flow, for maintaining an artificial diffusion term in the constitutive equation in order to guarantee numerical stability. Comparison with a DNS channel flow database [7] present coherent trends so far.

For further work, we intend to investigate the sensitivity of the square-root version of the code with respect to the artificial diffusivity, κ_b, by decreasing it even more. The evaluation of different drag reduction regimes by varying Re_τ, We_τ and L is another target.

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