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EVALUATING THE INFLUENCE OF POLYMERS ON THE IDENTIFICATION OF VORTICES

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Abstract. *Most of the vortex identification criteria are based on the kinematics of the flow. In the present work we examine the dynamic terms that are parts of a certain vortex identification criterion in order to decouple the influences of these terms. This methodology is applied to Newtonian and viscoelastic drag-reducing flow in a plane channel.*

Keywords: *flow classification, vortex identification, turbulence, DNS, channel flow*

1. INTRODUCTION

Vortex identification is still a non-consensual issue in Fluid Mechanics. In this connection, different criteria are used to identify a vortex. The most used criteria for vortex identification are based on kinematic quantities. These criteria would be useful if one wants to associate vortices with other physics of the problem, such as the ability of the flow to optimize diffusive or advective fluxes through the flow domain.

It is known that turbulent flows present ordered motion of vortices usually named *coherent structures*. These structures interact with each other and with the flow, and a better understanding of such interactions may lead to more accurate turbulent models and more precise control of turbulent processes as well. In the context of viscoelastic fluids, it is known that the addition of polymers in a Newtonian fluid can lead to a drag reduction in turbulent flows (Graham, 2004; White and Mungal, 2008) and this reduction is accompanied with a weakening of the turbulent structures (Stone *et al.*, 2004; Kim *et al.*, 2007, 2008; Kim and Sureshkumar, 2013).

In the present work, based on the criterion developed by Jeong and Hussain (1995), we are evaluating the influence of polymers diluted in a Newtonian fluid on the composition of the λ_2 -criterion from a dynamical viewpoint. The polymer contribution is taken into account by considering the Finite Extensible Non-linear Elastic model with the Peterlin approximation (FENE-P). We are using the DNS database provided by Thais *et al.* (2011, 2012), which encompasses Newtonian and viscoelastic simulations.

2. VORTEX IDENTIFICATION

2.1 The λ_2 -criterion

The λ_2 -criterion, originally proposed by Jeong and Hussain (1995), is based on the idea of local pressure minima. The authors claim that a good mathematical entity containing such information is the Hessian of the pressure, $\text{He}(p)$. Departing from the Navier-Stokes equation,

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{Re_h} \Delta \mathbf{u}, \quad (1)$$

the Hessian of the pressure may be found by applying the gradient operator to it, which yields

$$\nabla \left(\frac{\partial \mathbf{u}}{\partial t} \right) + \nabla (\mathbf{u} \cdot \nabla \mathbf{u}) = \nabla (-\nabla p) + \nabla \left(\frac{1}{Re_h} \Delta \mathbf{u} \right). \quad (2)$$

In Eq. (1), $Re_h = hU_b/\nu_0$ is the bulk Reynolds number, with h being a characteristic length scale, U_b the bulk velocity and ν_0 the kinematic viscosity.

At this point, Jeong and Hussain (1995) choose to look at the symmetric part of Eq. (2), because it contains the Hessian of the pressure and the terms that effectively contribute to it. This equation reads

$$\frac{D\mathbf{D}}{Dt} + \mathbf{D}^2 + \mathbf{W}^2 = -\text{He}(p) + \frac{1}{Re_h} \Delta \mathbf{D}, \quad (3)$$

and represents the evolution equation of the strain-rate tensor, \mathbf{D} .

Jeong and Hussain (1995) advocate that the unsteady and viscous term in Eq. (3) should be dropped because they lead to misleading results. More precisely, they present some specific cases in which unsteady flows identify vortices where are no pressure minimum, and others in which pressure minima are identified in regions without vorticity. With such assumptions, Eq. (3) becomes

$$\mathbf{D}^2 + \mathbf{W}^2 = -\text{He}(p), \quad (4)$$

and the Hessian of the pressure depends only on kinematic tensors.

According to Jeong and Hussain (1995), the mathematical condition for local pressure minima at the plane of vorticity is that the Hessian of the pressure must have two positive eigenvalues. If we reorder its eigenvalues so that $\lambda_1^{\text{He}(p)} \geq \lambda_2^{\text{He}(p)} \geq \lambda_3^{\text{He}(p)}$, then, it is sufficient to look at the sign of its intermediate eigenvalue, $\lambda_2^{\text{He}(p)}$. The authors propose, however, that one can use Eq. (4) to look at the sign of the intermediate eigenvalue of the tensor $\mathbf{D}^2 + \mathbf{W}^2$, $\lambda_2^{\mathbf{D}^2 + \mathbf{W}^2}$. Thus, according to the λ_2 -criterion, the condition for a vortex is

$$\lambda_2^{\text{He}(p)} > 0 \Leftrightarrow \lambda_2^{\mathbf{D}^2 + \mathbf{W}^2} < 0. \quad (5)$$

2.2 The inclusion of polymeric effects

In the FENE-P model, an extra-stress tensor containing tensions due to the presence of polymers is added to the Navier-Stokes equations, as follows

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{\beta}{Re_h} \Delta \mathbf{u} + \frac{1}{Re_h} \nabla \cdot \mathbf{\Xi}, \quad (6)$$

where β is the ratio of the Newtonian solvent viscosity (ν_N) to the total zero-shear viscosity ($\nu_0 = \nu_N + \nu_{p0}$), and the extra-stress tensor, $\mathbf{\Xi}$, is given by the relation

$$\mathbf{\Xi} = \frac{1-\beta}{Wi_h} (f(\text{tr}(\mathbf{c})) \mathbf{c} - \mathbf{I}), \quad (7)$$

in which \mathbf{c} is the conformation tensor representing the spatial configuration of polymer chains, and $Wi_h = \lambda U_b/h$ is the bulk Weissenberg number (λ being the relaxation time scale, U_b/h being a representative time scale).

The function f is given by the Peterlin function, which limits the maximum length of polymer chains to L , as follows

$$f(\text{tr}(\mathbf{c})) = \frac{L^2 - 3}{L^2 - \text{tr}(\mathbf{c})}. \quad (8)$$

Following Jeong and Hussain (1995) and taking the symmetric part of the gradient of Eq. (1), after some manipulation, we arrive at

$$\mathbf{D}^2 + \mathbf{W}^2 = -\frac{d\mathbf{D}}{dt} + \frac{\beta}{Re_h} \Delta \mathbf{D} - \text{He}(p) + \frac{1}{Re_h} \mathbf{S}^P, \quad (9)$$

where $\text{He}(p)$ is the Hessian of the pressure and \mathbf{S}^P is the symmetric part of $\nabla(\nabla \cdot \mathbf{\Xi})$.

Thus, when applying the λ_2 -criterion to a FENE-P-type fluid, the considered tensor is that on the sum of the left-hand side in Eq. (9). If we consider the same assumptions made by Jeong and Hussain (1995), the polymer contribution to the λ_2 -criterion is represented by the term $(1/Re_h) \mathbf{S}^P$.

It is worth noticing that, although the λ_2 -criterion is considered a kinematic criterion, mainly because it is calculated with the strain- and rotation-rate tensors, respectively \mathbf{D} and \mathbf{W} , this criterion is composed by dynamic entities (see the terms in Eq. (9)).

3. A DYNAMICAL VIEWPOINT FOR THE λ_2 -CRITERION

In order to verify how each dynamic term on the right-hand side in Eq. (9) contributes to the identification of a λ_2 -vortex, we work with the projection of Eq. (9) on the direction of the eigenvector ($e_2^{D^2+W^2}$) associated to the intermediate eigenvalue ($\lambda_2^{D^2+W^2}$) of the tensor $\mathbf{D}^2 + \mathbf{W}^2$. This is obtained by applying the following operation to Eq. (9)

$$\begin{aligned} \lambda_2^{D^2+W^2} = & e_2^{D^2+W^2} \cdot \left[-\frac{d\mathbf{D}}{dt} \right] \cdot e_2^{D^2+W^2} + e_2^{D^2+W^2} \cdot [-\text{He}(p)] \cdot e_2^{D^2+W^2} + \\ & e_2^{D^2+W^2} \cdot \left[\frac{\beta}{Re_h} \Delta \mathbf{D} \right] \cdot e_2^{D^2+W^2} + e_2^{D^2+W^2} \cdot \left[\frac{1}{Re_h} \mathbf{S}^P \right] \cdot e_2^{D^2+W^2}. \end{aligned} \quad (10)$$

Note that, on the left-hand side of Eq. (10), $\lambda_2^{D^2+W^2}$ is recovered and, on the right-hand side, the contribution of each of the dynamical terms make up the λ_2 -criterion.

By expressing the kinematic quantity associated to the λ_2 -criterion as a function of the dynamic terms that cause the motion, we are able to decouple the influence of the force-related quantities on the overall vortex structure. This approach can be particularly useful in the case of viscoelastic turbulence, since we can analyze the dynamical quantities responsible for the weakening of the turbulent structures (Stone *et al.*, 2004; Kim *et al.*, 2007, 2008; Kim and Sureshkumar, 2013) in drag-reducing flows.

4. RESULTS

We use here the instantaneous velocity and pressure fields that are outputs of DNS of plane channel flows performed by Thais *et al.* (2011, 2012) for Newtonian and viscoelastic fluids at a friction Reynolds number ($Re_{\tau 0} = hu_{\tau}/\nu_0$, where h is the channel half-gap and u_{τ} is the friction velocity) equal to 1000. By varying the maximum extensibility of the polymer chain (L) and its relaxation time (λ) – represented here in the form of the friction Weissenberg number, $Wi_{\tau 0} = \lambda u_{\tau}^2/\nu_0$ –, two levels of elasticity have been compared. The least elastic case ($L = 30$ and $Wi_{\tau 0} = 50$) provides a relative drag reduction of 30% whereas the most elastic one ($L = 100$ and $Wi_{\tau 0} = 115$) achieves 58%.

The mean contribution of each one of the dynamic terms in Eq. (10) in wall-parallel planes is presented in Fig. 1.

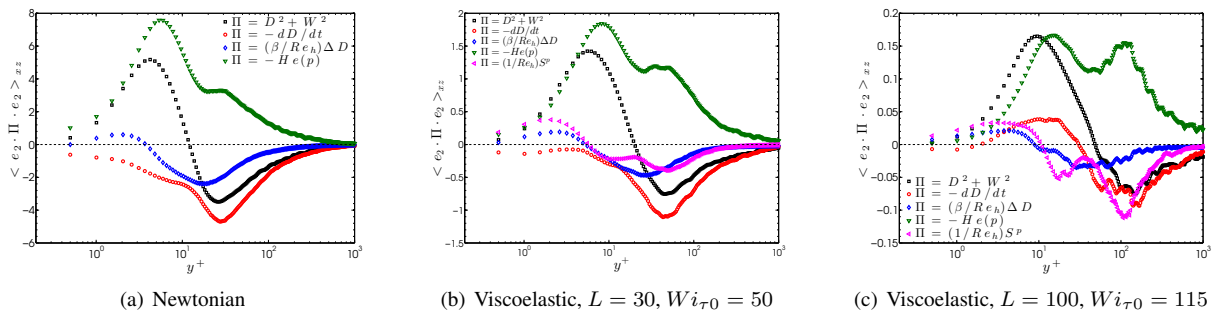


Figure 1. Contribution of terms in Eq. (10) for the identification of a λ_2 -vortex at $Re_{\tau 0} = 1000$.

Figure 1(a) contains the results for the Newtonian case. All terms tend to be null at the wall and at the channel centerline. The Hessian of the pressure is essentially positive, contributing in the sense of strain regions instead of vortex regions. Contrarily, the unsteady term ($d\mathbf{D}/dt$) is mostly negative, contributing in the sense of vortex regions. The viscous term contributes mostly negatively as well, with a slightly different behavior very near to the wall ($y^+ \lesssim 4$),

where it contributes positively. Consequently, λ_2 departs from zero at the wall to a maximum value at $y^+ \approx 5$. After this peak, it decreases to zero at $y^+ \approx 12$ and achieve a minimum close to $y^+ = 20$. After this valley, λ_2 tends to zero as it approaches the channel centerline.

It is remarkable that λ_2 is mostly dictated by the Hessian of the pressure in the vicinity of the wall and, away from the wall, the value of λ_2 basically equals the viscous term.

Regarding the viscoelastic cases (Figs. 1(b) and 1(c)), all terms follow similar trends comparing to the Newtonian case, except for some important points. In particular, the unsteady term has the tendency to become positive within the buffer layer ($5 \lesssim y^+ \lesssim 30$) with increasing elasticity. More importantly, two general features are evident. With increasing elasticity, the intensity of all terms decrease, and the maxima and minima occur further away with respect to the wall. The first point is another evidence of the well-known weakening of vortices in drag-reducing flows. However, we also highlight the weakening of strain-dominated regions (positive values). As far as the maxima and minima being shifted away from the wall, this may be related to the thickening of the buffer layer in viscoelastic flows. In fact, the two major theories on the drag reduction mechanism predicts this as a consequence of their explanations.

5. CONCLUSIONS

We evaluated the contribution of dynamic terms that contributes to the determination of vortices according to the λ_2 -criterion Jeong and Hussain (1995). Following Jeong and Hussain (1995), the symmetric part of the gradient of the momentum equation has been evaluated. Moreover, we include in this equation the contribution due to the presence of polymers. Instantaneous fields of turbulent channel flow of Newtonian and viscoelastic fluids at $Re_{\tau 0} = 1000$ generated by Thais *et al.* (2011, 2012) have been used for the analysis.

The present results suggests that the already known weakening of vortices in drag-reducing flows is not a direct effect of the polymer stress. Instead, it seems that it is a consequence of nonlinear interactions between the polymer stress and flow dynamics. Moreover, the weakening of vortices is also accompanied by a weakening of strain-dominated regions. Finally, the viscoelastic analyzes provides some evidences of the thickening of the buffer layer, which is predicted by the two major theories on the drag reduction phenomenon (Lumley, 1969; Tabor and de Gennes, 1986).

Finally, Jeong and Hussain (1995) relate the Q -criterion (Hunt *et al.*, 1988) with the λ_2 -criterion by the following equation

$$Q = -\frac{1}{2}\text{tr}(\mathbf{D}^2 + \mathbf{W}^2). \quad (11)$$

Therefore, the same methodology can be applied to the Q -criterion using Eq. (9) and is aimed as further work.

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7. REFERENCES

- Graham, M.D., 2004. “Drag reduction in turbulent flow of polymer solutions”. *Rheology Reviews*, pp. 143–170.
- Hunt, J.C.R., Wray, A.A. and Moin, P., 1988. “Eddies, stream, and convergence zones in turbulent flows”. *Center for Turbulence Research - Proceedings of Summer Program*, Vol. Report CTR-S88, pp. 193–208.
- Jeong, J. and Hussain, F., 1995. “On the identification of a vortex”. *J Fluid Mech*, Vol. 285, pp. 69–94.
- Kim, K., Adrian, R.J., Balachandar, L. and Sureshkumar, R., 2008. “Dynamics of hairpin vortices and polymer-induced turbulent drag reduction”. *Physical Review Letters*, Vol. 100, pp. 134504–1–134504–4.
- Kim, K., Li, C.F., Sureshkumar, R., Balachandar, L. and Adrian, R.J., 2007. “Effects of polymer stresses on eddy structures in drag-reduced turbulent channel flow”. *J Fluid Mech*, Vol. 584, pp. 281–299.
- Kim, K. and Sureshkumar, R., 2013. “Spatiotemporal evolution of hairpin eddies, reynolds stress, and polymer torque in polymer drag-reduced turbulent channel flows”. *Phys. Rev. E*, Vol. 87, p. 063002. doi:10.1103/PhysRevE.87.063002. URL <https://link.aps.org/doi/10.1103/PhysRevE.87.063002>.
- Lumley, J.L., 1969. “Drag reduction by additives”. *Annu Rev Fluid Mech*, Vol. 11, pp. 367–384.
- Stone, P.A., Roy, A., Larson, R.G., Waleffe, F. and Graham, M.D., 2004. “Polymer drag reduction in exact coherent structures of plane shear flow”. *Physics of Fluids*, Vol. 16, No. 9, pp. 3470–3482. doi:<http://dx.doi.org/10.1063/1.1775192>. URL <http://scitation.aip.org/content/aip/journal/pof2/16/9/10.1063/1.1775192>.
- Tabor, M. and de Gennes, P.G., 1986. “A cascade theory of drag reduction”. *Europhys Lett*, Vol. 7, pp. 519–522.
- Thais, L., Gatski, T.B. and Mompean, G., 2012. “Some dynamical features of the turbulent flow of a viscoelastic fluid for reduced drag”. *Journal of Turbulence*, Vol. 13, pp. 1–26.

- Thais, L., Tejada-Martinez, A., Gatski, T.B. and Mompean, G., 2011. "A massively parallel hybrid scheme for direct numerical simulation of turbulent viscoelastic channel flow". *Comput Fluids*, Vol. 43, pp. 134–142.
- White, C.M. and Mungal, M.G., 2008. "Mechanics and prediction of turbulent drag reduction whit polymer additives". *Annual Review of Fluid Mechanics*, Vol. 40, pp. 235–256.

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