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EVALUATION OF THE STATISTICAL CONVERGENCE OF TURBULENT FLOW DATA

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Abstract. *Direct numerical simulations (DNS) provide useful information for the understanding and the modeling of turbulent phenomena. Particularly, in the context of turbulence closure models for computational fluid dynamics, e.g. Reynolds Averaged Navier–Stokes Equations (RANS) and Large Eddy Simulation (LES), DNS has been largely used to supply contributions to the understanding of the flow behavior at small scales. However, quantifying the errors associated with DNS is an important challenge to the scientific community. Recently, Thompson et al. (Computers & Fluids, 2016) presented a methodology to evaluate the statistical errors of the second-moment DNS data of Newtonian fluid flows, where the steady-state momentum balance equation was used to calculate the mean velocity profile by considering the Reynolds stress provided by DNS data. The authors applied the methodology to several channel flow databases available in the literature. In the present work, we follow the main idea of Thompson et al. (Computers & Fluids, 2016) to evaluate the statistical errors of Newtonian fluid channel and pipe flows. The present analysis was made by using data generated with the quasi-spectral code by Thais et al. (Computers & Fluids, 2011) for several friction Reynolds number. The simulation time necessary to reach the statistically converged fields is characterized as a function of the Reynolds number, comparisons with other authors results are established. The present methodology could be used as a convergence criteria to future DNS of Newtonian fluids and to provide more accurate benchmark results for turbulence modeling.*

Keywords: *direct numerical simulation, statistical convergence, turbulence, statistic errors, plane channel flow.*

1. INTRODUCTION

In previous work, Thompson et al. (2016) presented an approach to estimate the statistical error associated with the Reynolds stress tensor provided by direct numerical simulation (DNS) of turbulent plane channel flow. As noticed by Vinuesa et al. (2016), there is a lack of consensus in the community about the issue of statistical convergence in DNS. Herein we extend the method proposed by Thompson et al. (2016) to characterize and estimate the statistical convergence of uncertainties with respect to averaging time. Comparison of this convergence criterion is made between results of the present work and DNS plane channel flow and pipe flow databases available in the literature.

The main database parameters from other authors for the different Reynolds numbers used in this work are given by Tabs. 1 and 2.

Table 1. Overview of publicly accessible DNS databases at $Re_\tau \approx 180$.

Author	Abbreviation	Method	Re_τ	$T\,u_\tau/h$	T^*
Present Study	PS	F-FD6	180	235	1544
Bernardini *et al.* (2014)	BPO	FD2-FD2	183	-	-
Del Álamo and Jiménez (2003).	AJ	F-C	186	50	1315
Eggels *et al.* (1994)	EUWWAFN	FV2-FV2	180	28	46
Fukagata and Kasagi (2002)	FK	FD2-FD2	180	11	36
Khoury *et al.* (2013)	KSNFBJ	SEM-SEM	181	-	-
Lee and Moser (2015)	LM	FG-BS7	182	32	421
Moser *et al.* (1999)	MKM	F-C	178	-	-
Vreman and Kuerten (2014a)	VK-FD2	FD2-FD2	180	200	585
Vreman and Kuerten (2016)	VK-S2B3	F-C	180	200	585
Vreman and Kuerten (2016)	VK-S4B3	F-C	180	100	146
Wu and Moin (2008)	WM	FD2-FD2	181	19	47

Table 2. Overview of publicly accessible DNS databases at $Re_\tau \approx 590$.

Author	Abbreviation	Method	Re_τ	$T\,u_\tau/h$	T^*
Present Study	PS	F-FD6	590	97	640
Abe *et al.* (2004)	AKC	FD4-FD2	640	10	44
Bernardini *et al.* (2014)	BPO	FD2-FD2	550	36	236
Del Álamo and Jiménez (2003).	AJ	F-C	547	12	210
Iwamoto *et al.* (2002)	ISK	F-C	642	40	55
Fukagata and Kasagi (2002)	FK	FD2-FD2	550	-	-
Lee and Moser (2015)	LM	FG-BS7	544	14	184
Moser *et al.* (1999)	MKM	F-C	587	-	-
Thais *et al.* (2011)	TGM	F-FD	590	20	131
Vreman and Kuerten (2016)	VK	F-C	590	100	109
Wu *et al.* (2012)	WBA	FD2-FD2	685	14	70

2. METHODOLOGY

In steady-state fully-developed channel flow, the total shear stress τ is a linear function of wall distance y. It is also known that τ is constrained by the boundary conditions $\tau = \tau_w$ at the wall and $\tau = 0$ at the center of the geometry. At the same time, we can split the total shear stress, τ, as the sum of the molecular shear stress and of the Reynolds shear stress $Re_{xy} = \overline{-u_1' u_2'}$. Using $u_\tau = \sqrt{\tau_w/\rho}$ as characteristic velocity and $\delta_\nu = \nu/u_\tau$ as characteristic length, we get:

$$\left(1 - \frac{y^+}{Re_\tau}\right) = \frac{dU^+}{dy^+}(y^+) + R_{xy}^+(y^+),, \quad (1)$$

where U is the mean velocity in the streamwise x-direction, y is the wall direction and ρ is the mass density. Variables with a superscript +, are expressed in wall units.

In the present work, Eq. 1 is used as means to discuss the consequences of the numerical imbalance of the fully developed momentum equation. We employ the symbol $\widehat{()}$ to label any variable that is provided by the DNS hybrid space-time average or experimental result. The symbol $\widetilde{(\,)}$ is used to label a variable that is computed from a conservative equation such as Eq. 2. With this convention, we can write the following two equations

$$\tilde{R}^+_{xy}(y^+) = 1 - \frac{y^+}{Re_\tau} - \frac{d\hat{U}}{dy^+}(y^+), \tag{2}$$

$$\tilde{U}^+_{xy}(y^+) = y^+ - \frac{{y^+}^2}{2Re_\tau} - \int_0^{y^+} \hat{R}^+_{xy}(y')dy'. \tag{3}$$

Eq. 2 expresses the yx-component of the Reynolds stress tensor that balances the momentum equation in the x-direction using the axial mean velocity gradient provided by input data. On the other hand, Eq. 3 expresses the mean velocity profile that would balance the momentum equation using the yx-component of the Reynolds stress tensor available from input data.

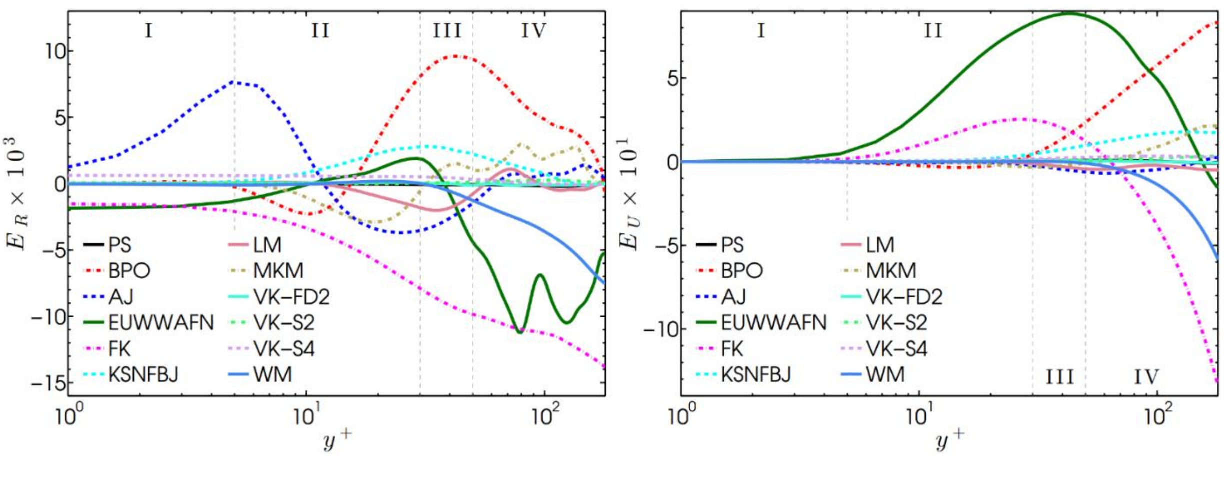
Equations 2 and 3 can be used to define two residuals $E_R\,(y^+) = \tilde{R}^+_{yx}\,(y^+) - \hat{R}^+_{yx}\,(y^+)$ and $E_U(y^+) = \tilde{U}^+_{yx}(y^+) - \hat{U}^+_{yx}(y^+)$, which can be interpreted as the residual of the momentum balance with respect to steady state fully developed flow. To characterize the convergence of input statistics depending on the averaging time, a residual criteria independent of y and varying on T is considered:

$$\|E_R\| = \frac{1}{Re_\tau}\int_0^{Re_\tau}[E_R(y^+)]^2\,dy^+, \tag{4}$$

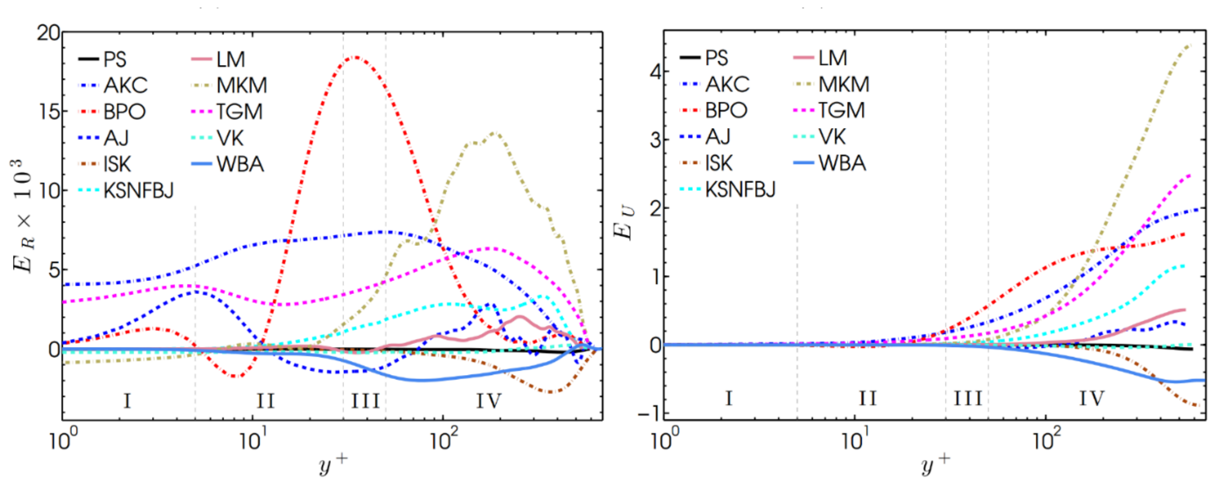
$$\|E_U\| = \frac{1}{Re_\tau}\int_0^{Re_\tau}[E_U(y^+)]^2\,dy^+. \tag{5}$$

3. RESULTS AND DISCUSSION

Figure 1 displays the residuals $E_R\,(y^+)$ and $E_U\,(y^+)$ across the channel half-width. Singular patterns can be identified for each dataset. For example, for Bernardini et al. (2011, 2014) the maximum E_R is found to be in region III. For Álamo and Jiménez (2003) and Álamo et al. (200), the peak is located on the line separating the viscous sub-layer (regions I) and buffer layer (region II).



(a) E_R at $Re_\tau \approx 180$ (b) E_U at $Re_\tau \approx 180$

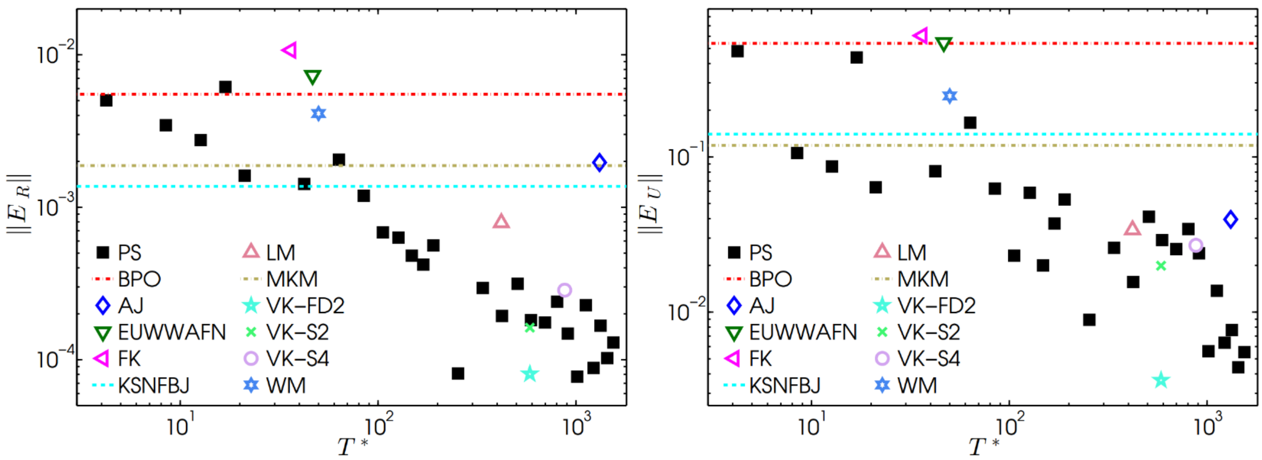


(c)E_R at $Re_\tau \approx 544-685$ (d)E_U at $Re_\tau \approx 544-685$

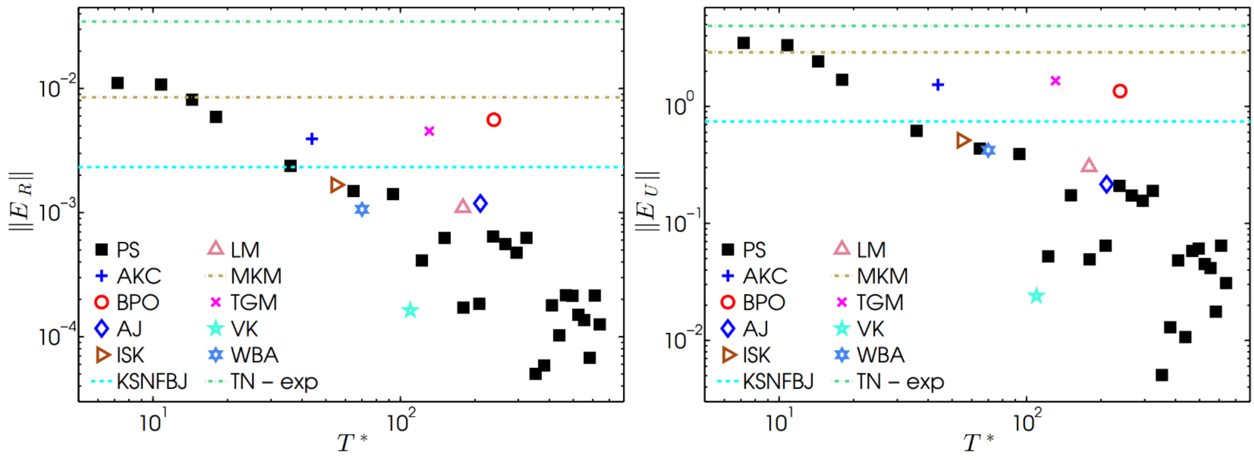
Figure 1: Comparison of residuals values (E_R (y^+) and E_U (y^+)) between the present study and several DNS databases.

Figure 2 shows the temporal development of the residual norms $\|E_R\|$ and $\|E_U\|$. Results are shown in function of the averaging non-dimensional time T^*, given by Vinuesa et al. (2016). The non-dimensional average time T^* is given by $T(u_\tau/h)(L_xL_z/18h^2)$, where L_x, L_z are the stream-wise and span-wise box lengths and $2h$ is the channel height.

In the beginning of calculations, the errors decay faster and its convergence rate decreases along the averaging time. Some authors that in principle used the same averaging time, reached very different convergence levels and in other hand, authors with different averaging time reached almost the same error, it confirms that other parameters are also very important to the statistical convergence, e.g., time step, box size, spatial and temporal discretization method, mesh quality, etc.



(a)$\|E_R\|$ at $Re_\tau \approx 180$ (b)$\|E_U\|$ at $Re_\tau \approx 180$



(c) $\|E_R\|$ at $Re_\tau \approx 544 - 685$ (d) $\|E_U\|$ at $Re_\tau \approx 544 - 685$

Figure 2: Development of the residuals norm $\|E_R\|$ and $\|E_U\|$ as function of the averaging time. Plotted horizontal lines report data with unidentified averaging time.

4. CONCLUSION

The present work develops and investigates a novel convergence criterion to of turbulent plane channel flow. From the perspective of the momentum balance equation or its integrated form, the mean streamwise velocity and Reynolds stress tensor can be interpreted as dual statistical quantities. Considering one of this quantities as a given, the other can be computed. The deviation between the computed value and the corresponding provided value can be considered an error estimation for the evaluated statistical quantity.

The convergence rates show to have a strong dependence on the averaging time, but others parameters play important roles on the statistical convergence, e.g., initial time to start averages, time step, box size, etc. Some of these parameters have heuristically chosen values and are often omitted by the authors.

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